

The soft Pomeron from AdS-CFT

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Abstract

In previous treatments, high energy QCD was analyzed using AdS-CFT a la Polchinski-Strassler. Black hole production in AdS was responsible for power law behaviour of the total QCD cross section. Using the simplest self-consistent gravity dual assumption, that cut-off AdS_5 is supplemented by a 5d space X_5 of effective “average” size much larger than the scale of AdS_5 , we find an energy behaviour just before the saturation of the Froissart bound that is $\sigma_{tot} \sim s^{1/n} = s^{1/11} \simeq s^{0.0909}$. It comes from the solution of the Laplacean on $AdS_{d+1} \times X_{\bar{d}}$ behaving like $1/r^n = 1/r^{2(d-1)+\bar{d}} = 1/r^{11}$. We argue that this should be present in real QCD as well, as string corrections to the dual scattering are small, and should onset at about $N_c^2 M_{1,glueball} \sim 10 GeV$. Experimentally, one found the “soft Pomeron” behaviour, $\sigma_{tot} \sim s^{0.093(2)}$, that onsets at about $9 GeV$, that was later argued to be replaced by the unitarized Froissart + reaction-dependent constant behaviour. We argue that the soft Pomeron and the dual behaviour represent the same physics, creation of an effective field theory “soliton”-like structure (=black hole), that then decays, and so they have to be taken seriously. We thus have an experimental prediction of string theory, literally counting the extra dimensions.

1 Introduction

At very large center of mass energies \sqrt{s} (much larger than the hadrons mass, e.g. $\gg 1\text{GeV}$ for protons), the scattering of two hadrons in QCD is behaving in a “soft” manner: The total cross-section σ_{tot} for the large s , fixed t scattering has a slow dependence with s .

But there is a bound on $\sigma_{tot}(s)$ at large energies due to Froissart [1], with saturation of the type

$$\sigma_{tot} \sim \frac{A}{M^2} \ln^2 \frac{s}{s_0} \quad (1.1)$$

where M is the mass lightest of the lightest excitation in the theory and A is a constant satisfying $A \leq \pi$. In pure Yang-Mills, M is the mass of the lightest glueball excitation $M_{1,glueball}$, and if there is an almost Goldstone boson of smaller mass, like the pion of QCD, then $M = m_\pi$ and $A/M^2 \leq 60\text{mb}$.

Experimentally, one first found the “soft Pomeron” behaviour [2] (cited in the 2001 PDG [3]), with $\sigma_{tot} \sim s^{0.093(2)}$. More precisely, in the scattering of two hadrons A and B , after subtracting C -odd and C -even meson exchanges, one finds

$$\begin{aligned} \sigma_{AB} - Y_{1AB}(s_1/s)^{\eta_1} + Y_{2AB}(s_1/s)^{\eta_2} &= X_{AB}(s/s_0)^\epsilon \\ \sigma_{\bar{A}B} - Y_{1AB}(s_1/s)^{\eta_1} - Y_{2AB}(s_1/s)^{\eta_2} &= X_{AB}(s/s_0)^\epsilon \end{aligned} \quad (1.2)$$

and experimentally, one finds a $\chi^2/d.o.f. = 1$ fit for energies above $\sqrt{s_{min}} = 9\text{ GeV}$ (if we extend the fit to lower energies, $\chi^2/d.o.f.$ increases), giving $\epsilon = 0.0933 \pm 0.0024$ and $X_{AB} \sim 10 - 35\text{mb}$ [2].

However, later it was found [4] (cited in the 2004 PDG [5]) that the fit is better (with $\chi^2/d.o.f. = 0.971$) if we replace the “soft Pomeron” by the maximal Froissart behaviour, plus a constant, reaction-dependent term, i.e.

$$\begin{aligned} \sigma_{AB} - Y_{1AB}(s_1/s)^{\eta_1} + Y_{2AB}(s_1/s)^{\eta_2} &= Z_{AB} + B \log^2(s/s_0) \\ \sigma_{\bar{A}B} - Y_{1AB}(s_1/s)^{\eta_1} - Y_{2AB}(s_1/s)^{\eta_2} &= Z_{AB} + B \log^2(s/s_0) \end{aligned} \quad (1.3)$$

in which case the behaviour can be extended down to $\sqrt{s_{min}} = 5\text{GeV}$, with $B = 0.31\text{mb} \ll 60\text{mb}$ and $Z_{AB} \sim 18 - 65\text{mb}$.

Using AdS-CFT [6] for a general non-conformal theory, in [7] the high energy behaviour of gauge theories was analyzed, and power law behaviours for $\sigma_{tot}(s)$ were found, corresponding to black hole production in the gravity dual, that settle into the maximal Froissart behaviour. The saturation of the Froissart bound was proven, a fact which is still not done in QCD. Moreover, in [8] the saturation behaviour in the gravity dual was mapped exactly onto a 1952 effective field theory model due to Heisenberg [9], of collisions of shockwaves of pion field distributions.

In this paper, we will argue that one needs an extra assumption about the gravity dual, and then we find a power law $\sigma_{tot} \sim s^{1/11}$ setting in at about $N_c^2 M_{1,glueball}$, which is around 10 GeV in QCD, and that will settle into the maximal Froissart behaviour. Thus we argue that the “soft Pomeron” behaviour is real, gives a string theory prediction, and one needs further experimental work to determine how does the argued-for maximal Froissart behaviour in [4] fit in. For previous attempts at describing the soft Pomeron in AdS-CFT, see [10].

We will first explain and expand on the large N , large $g^2 N$ analysis of high energy gauge theory scattering in [7] (section 2), then explain the $s^{1/11}$ behaviour and summarize the energy regimes of gauge theories (section 3). Then we will describe what happens in real QCD and compare to the experimental evidence (section 4).

2 Using AdS-CFT duality for high energy gauge theory scattering

Polchinski and Strassler [11] (see also [12]) have shown that one can describe the scattering of colourless states at large energies in a gauge theory by scattering in a minimal model of gravity dual. A conformal theory is dual to an $AdS_5 \times X_5$ space

$$ds^2 = \frac{\bar{r}^2}{R^2} d\vec{x}^2 + \frac{R^2}{\bar{r}^2} d\bar{r}^2 + R^2 ds_X^2 = e^{-2y/R} d\vec{x}^2 + dy^2 + R^2 ds_X^2 \quad (2.1)$$

and to describe a nonconformal theory one just cuts off the space in the IR, at $\bar{r}_{min} \sim R^2 \Lambda_{QCD}$ (equivalently, at y_{max}), where Λ_{QCD} is the mass of the lightest excitation of the gauge theory (glueball). This hides our ignorance about what happens in the IR, corresponding to small r modifications of the gravity dual, but this simple model is enough to obtain many features of the gauge theory scattering. This cut off is equivalent to putting an IR brane, thus getting the Randall-Sundrum model [13] (if we put an optional UV cut off).

A gauge theory mode with momentum p and wavefunction e^{ipx} corresponds in the gravity dual to a mode with local AdS momentum $\tilde{p}_\mu = (R/\bar{r})p_\mu$ and wavefunction $e^{i\tilde{p}x}\psi(\bar{r}, \Omega)$, and the string tension $\alpha' = R^2/(g_s N)^{1/2}$ corresponds to the gauge theory string tension $\hat{\alpha}' = \Lambda_{QCD}^{-2}/(g_{YM}^2 N)^{1/2}$ and $\sqrt{\alpha'}\tilde{p}_{string} \leq \sqrt{\hat{\alpha}'}q_{QCD}$. The gauge theory amplitudes are related to string amplitudes by

$$\mathcal{A}_{gauge}(p) = \int d\bar{r} d^5\Omega \sqrt{g} \mathcal{A}_{string}(\tilde{p}) \prod_i \psi_i \quad (2.2)$$

At large \bar{r} , ψ behaves as

$$\psi(\bar{r}, \Omega) \sim C(\bar{r}/\bar{r}_{min})^{-\Delta} g(\Omega) \quad (2.3)$$

Since $\mathcal{A}_{string} = \mathcal{A}_{string}(\tilde{s}, \tilde{t})$, one takes $\nu = -\alpha'\tilde{t}$ as integration variable (then $\bar{r} = \nu^{-1/2}\bar{r}_{min}\sqrt{\hat{\alpha}'t}$), with $\mathcal{A}_{string} = \mathcal{A}_{string}(\nu s/|t|, \nu)$. If one takes the large \bar{r} behaviour of the wavefunctions to be valid everywhere, one finds that most of the integration in the high energy ($s \rightarrow \infty$) case is situated in the IR (small \bar{r}). However, as we can easily see, the fact that the wavefunction gets modified at small \bar{r} will only modify the behaviour of \mathcal{A}_{gauge} with t , the s behaviour still comes from \mathcal{A}_{string} . What can also happen is that the modification of the wavefunction is such as to keep the bulk of the integral centered not on \bar{r}_{min} , but a finite distance away from it.

Giddings then noticed that one will produce black holes in the gravity dual when one reaches the Planck scale $M_P = g_s^{-1/4} \alpha'^{-1/2} = N^{1/4} R^{-1}$, corresponding to the gauge theory scale $\hat{M}_P = N^{1/4} \Lambda_{QCD}$ [14]. The black hole horizon radius in flat D dimensions grows with

mass like $r_H \sim M^{1/(D-3)}$, thus if the cross section for black hole formation is assumed to be a simple black disk with radius $r_H(M = \sqrt{s})$, the cross section for black hole formation will grow like

$$\sigma \sim \pi r_H^2 \sim (\sqrt{s})^{\frac{2}{D-3}} \quad (2.4)$$

For $D=5$ (only AdS is approximately flat), $\sigma \sim s^{1/2}$, whereas if $D=10$ ($AdS_5 \times S_5$ is approximately flat), $\sigma \sim s^{1/7} \simeq s^{0.143}$.

As the black holes grow in size, the horizon of the 10d black hole will reach the AdS size when $E = E_R = M_P(RM_P)^7 = N^2 R^{-1}$, corresponding in gauge theory to $\hat{E}_R = N^2 \Lambda_{QCD}$, and this was argued that should correspond to the maximal Froissart behaviour in gauge theory. Indeed, if one takes the linearized gravity induced by a point mass $m = \sqrt{s}$ on the IR brane, and obtains the horizon radius for it by setting the perturbation to 1, one obtains

$$h_{00,lin} \sim G_4 \sqrt{s} \frac{e^{-M_1 r}}{r} \sim 1 \Rightarrow \sigma \sim \pi r_H^2 \sim \frac{\pi}{M_1^2} \ln^2(\sqrt{s} G_4 M_1) \quad (2.5)$$

which with the assumption $\sigma_{QCD} \sim \sigma$ is just the maximal Froissart behaviour, in the case the mass M is the lightest glueball mass, corresponding to the lightest KK graviton in the dual, $M_1 = j_{1,1}/R$. Then indeed, $r_H \sim 1/M_1 \sim R$.

The case when the pion (almost Goldstone boson) is lightest is treated similarly, by making the radion of the Randall Sundrum model dynamical, with mass M_L . Then the IR brane bends under the mass, and now

$$\frac{\delta L}{L}|_{lin} \sim G_4 \sqrt{s} (M_L R) \frac{e^{-M_L r}}{r} \sim 1 \Rightarrow \sigma_{QCD} = \sigma \sim \frac{\pi}{M_L^2} \ln^2(\sqrt{s} G_4 M_L R) \quad (2.6)$$

and one gets the Froissart bound with $M_L \leftrightarrow m_\pi$.

This simple analysis was made more rigorous and exact (i.e. calculating coefficients) in [7] and was mapped exactly to the Heisenberg model [9] for the saturation in [8]. This was done as follows.

The scattering at high energies in the gravity dual can be described by scattering of Aichelburg-Sexl shockwaves [15] in the gravity dual. In flat D dimensions, the shockwaves are

$$ds^2 = 2dx^+ dx^- + (dx^+)^2 \Phi(x^i) \delta(x^+) + d\vec{x}^2 \quad (2.7)$$

where the function Φ satisfies the Poisson equation

$$\Delta_{D-2} \Phi(x^i) = -16\pi G p \delta^{D-2}(x^i) \quad (2.8)$$

One can put these shockwaves inside gravity duals [16], and one takes advantage of the fact that one still has the Poisson equation (2.8) for the function Φ , just the Laplacean is taken in the background. Thus for the shockwaves, the linearized solution is the exact solution, and one can find it explicitly, unlike the black hole solution in the background. For early shockwave-type solutions in AdS and AdS-CFT see [17], used to argue for black hole creation [18]. The linearization of shockwaves in AdS and on braneworlds was observed in [19].

At energies below the Planck scale M_P (in gauge theory $\hat{M}_P = N^{1/4} \Lambda_{QCD}$), or rather below the string scale $E_s = \alpha'^{-1/2}$ (in gauge theory $\hat{E}_s = (g_{YM}^2 N)^{1/4} \Lambda_{QCD}$), one takes only

one of the scattered particles as a shockwave, and the second as a null geodesic scattering in the background [20]. One can calculate this 't Hooft scattering in gravity duals [16], and in [7] the behaviour of the AdS amplitude $\mathcal{A}_{string} \sim G_4 s$ was found, giving

$$\frac{d\sigma_{AdS}}{d^2k} = \frac{4}{s} \frac{d\sigma_{AdS}}{d\Omega} \sim (G_4 s)^2 \quad (2.9)$$

and correspondingly in gauge theory

$$\mathcal{A}_{gauge} \sim \hat{G}_4 s \Rightarrow \frac{d\sigma_{gauge}}{d^2k} = \frac{4}{s} \frac{d\sigma_{gauge}}{d\Omega} \sim (\hat{G}_4 s)^2 \quad (2.10)$$

At energies above the string scale $E_s = \alpha'^{-1/2}$ (in gauge theory $\hat{E}_s = (g_{YM}^2 N)^{1/4} \Lambda_{QCD}$), one has Regge behaviour of the string amplitude, which implies Regge behaviour for the gauge amplitude (from (2.2))

$$\mathcal{A}_{gauge}^{2 \rightarrow 2} \sim (\hat{\alpha}' s)^{2 + \hat{\alpha}' t/2} \quad (2.11)$$

At energies above the Planck scale M_P , one has to take both particles that scatter as shockwaves. The metric in the interacting region can only be calculated perturbatively away from the interaction point [21], but luckily one can calculate the presence of a “trapped surface” at the interaction point, and by a GR theorem there will be a horizon forming outside it, away from the interaction. One can calculate the trapped surface at nonzero impact parameter b , and derive a maximum $b_{max}(s)$ for which a trapped surface forms. This formalism was put forward in flat 4d in [22] and generalized to curved higher d in [23], with an approximation scheme for $b_{max}(s)$.

Once we have a $b_{max}(s)$ describing this classical scattering, one can use a simple eikonal model to get a quantum amplitude from it, with the eikonal being the simplest thing we can have, a black disk

$$Re(\delta(b, s)) = 0 \quad Im(\delta(b, s)) = 0, \quad b > b_{max}(s); \quad Im(\delta(b, s)) = \infty, \quad b < b_{max}(s) \quad (2.12)$$

Then the resulting quantum amplitude can be put in the Polchinski-Strassler formula (2.2) to derive a gauge amplitude, and the result is that the classical gravity σ_{tot} gets multiplied by a model-dependent constant (depends on the details of the gravity dual, that we have approximated by the RS model), and we replace R by Λ_{QCD} and M_P by $N^{1/4} \Lambda_{QCD}$ (gravity parameters replaced by gauge parameters).

This scattering model looks very much like Heisenberg’s model [9], with an exact match for the Froissart saturation, as shown in [8]. Heisenberg argues that at high energies, the hadrons scattering are replaced by pion field distributions that look like shockwaves, because of very large (infinite) Lorentz boosts. While Heisenberg had pion fields, in the case we are describing, of only Yang-Mills, we have instead the lightest glueball field, mapped to gravity excitations in the dual. Indeed, in the gravity dual description, we have collisions of gravity field shockwaves.

The nonlinearity of the pion field, described by Heisenberg through the DBI-like action

$$S = l^{-4} \int d^4x \sqrt{1 + l^4 [(\partial_\mu \phi)^2 + m^2 \phi^2]} \quad (2.13)$$

and in our case by the nonlinearity of the gravitational action in the dual (and of the glueball field in the gauge theory), is responsible for creating a nonlinear “soliton” in the collision. One cannot find it in perturbation theory (Heisenberg presents the perturbative pion solution), as seen also in our case: one can’t find the black hole in perturbation theory for the A-S collision [21]. The soliton will decay through emission of pions in Heisenberg’s model, and emission of gravitons for the black hole gravity dual.

For Heisenberg’s model, one has at $x^+ \leq 0, x^- \leq 0$,

$$\phi = \phi_1 + \phi_2 = \delta(x^+) \psi_1(x^i) + \delta(x^-) \psi_2(x^i) \quad (2.14)$$

where x^i are transverse coordinates, and the evolution of this shockwave should give the “soliton” at $x^+ > 0, x^- > 0$.

What brings in the saturation of the Froissart bound is the assumption that $\psi(x^i) \sim e^{-m_\pi r}$ and the “degree of inelasticity” $\alpha (= \mathcal{E}/\sqrt{s} = \text{energy loss/collision energy})$ behaves similarly to ψ as a function of the impact parameter b (see also [8] for a more detailed account).

But if we don’t make this extra assumption, we get an effective field theory model in the gauge theory for all black hole formation in the gravity dual.

Indeed, in the gravity dual, we scatter two A-S shockwaves, and for $M_P < \sqrt{s} < E_R$ (corresponding in gauge theory to $\hat{M}_P = N^{1/4} \Lambda_{QCD} < \sqrt{s} < \hat{E}_R = N^2 \Lambda_{QCD}$), the black holes formed can be considered to be in flat space. Correspondingly, we take A-S shockwaves in flat space, thus solutions to (2.8) in flat D-dimensional space, for which ($r = \sqrt{x^i x^i}$)

$$\Phi = \frac{16\pi G_D}{\Omega_{D-3}(D-4)r^{D-4}} \sim \frac{1}{r^{D-4}} \quad (2.15)$$

Then one obtains the maximum b for black hole formation [7]

$$\begin{aligned} b_{max}^2 &\leq 2 \left[\frac{\alpha}{D-2} \right]^{\frac{1}{D-3}} \frac{D-3}{D-2} = (D-3) \frac{2(\epsilon r_H)^2}{[D-2]^{\frac{D-2}{D-3}}} \\ r_H &= \left[\frac{16\pi G_D \sqrt{s}}{(D-2)\Omega_{D-2}} \right]^{\frac{1}{D-3}} \quad \epsilon = \left[\frac{(D-2)\Omega_{D-2}}{4\Omega_{D-3}} \right]^{\frac{1}{D-3}} \end{aligned} \quad (2.16)$$

and therefore $b_{max}(s) \simeq a s^{\frac{1}{2(D-3)}}$. Then using (2.2) the gauge theory amplitude is

$$\sigma_{gauge} = \bar{K} \pi a^2 \left(\frac{\hat{\alpha}' s}{\alpha'} \right)^{\frac{1}{D-3}} \quad (2.17)$$

where \bar{K} is a model dependent constant and in a we need to replace G_D by \hat{G}_D (gauge theory quantity). Again, for $D=5$ we get $\sigma \sim s^{1/2}$, and for $D=10$ we have $\sigma \sim s^{1/7} \simeq s^{0.143}$, but now we have an exact picture.

Now we see that we can also map to Heisenberg’s model, if we only relax the assumption about the form of $\psi(x^i)$ (which was natural for a pion of mass m_π), and say that $\psi(x^i)$ is instead mapped to $\Phi \sim 1/r^{D-4}$. Of course, the caveat is that the gravity dual picture is intrinsically D-dimensional, whereas the Heisenberg model is in 4d, but this is just the usual holography.

For the maximal Froissart behaviour however, the Heisenberg and dual descriptions match exactly. When the horizon size of the formed black holes becomes comparable with the AdS size, namely at $E = E_R$ in AdS and $\hat{E}_R = N^2 \Lambda_{QCD}$ is gauge theory, we have to consider the curvature of space into account. But as we saw, most of the integral in the Polchinski-Strassler formula (2.2) is situated at the IR end (IR brane), if the large \bar{r} behaviour of the wavefunctions remains (or is not modified too much). In that case, the black holes being mostly created near the IR brane will eventually be large enough not to feel they are away from the IR brane.

One takes then the scattering of two A-S shockwaves on the IR brane, that behave at large radius as

$$\Phi(r, y = 0) \simeq R_s \sqrt{\frac{2\pi R}{r}} C_1 e^{-M_1 r}; \quad C_1 = \frac{j_{1,1}^{-1/2} J_2(j_{1,1})}{a_{1,1}}; \quad J_1(z) \sim a_{1,1}(z - j_{1,1}); \quad z \rightarrow j_{1,1} \quad (2.18)$$

and one sees the same behaviour as one had for the static black hole perturbation $h_{00,lin.}$, namely the exponential drop, just the power of r and constants are different. This shockwave is the solution of the laplacean for a massless particle on the IR brane, and now satisfies exactly Heisenberg's description, and is also a 4d picture (the higher dimensional gravity was in a sense KK reduced for this solution, which lives on the IR brane. The parameter $M_1 = j_{1,1}/R$ is the first KK mass).

From the scattering of two such waves one finds the maximum impact parameter that forms a black hole,

$$b_{max}(s) = \frac{\sqrt{2}}{M_1} \ln[R_s M_1 K]; \quad K = \frac{3\sqrt{\pi}}{\sqrt{2} j_{1,1}^{3/2}} \simeq 0.501 \quad (2.19)$$

where $R_s = G_4 \sqrt{s}$, $G_4 = 1/(RM_{P,5}^3)$, $M_1 = j_{1,1}/R$ ($j_{1,1} \simeq 3.83$) and the gauge theory cross section is

$$\sigma_{gauge} = \bar{K} \pi b_{max}^2(\tilde{s}) \quad (2.20)$$

where as before we must replace gravity quantities with gauge theory quantities, and \bar{K} is a model dependent constant.

In [7] a possible intermediate case was analyzed also, when the black holes that are formed start feeling the AdS size, but not the IR brane yet. In that case, at large r , an A-S shockwave inside AdS_5 was found to behave like

$$\Phi = \frac{\bar{C} R^4}{r^6} e^{(4y+2y_0)/R} \quad (2.21)$$

where $\bar{C} = 2R_s R^2$ and the shockwave lives at $y = y_0$ in AdS_5 . Then in the scattering of two such waves, one finds the maximum b for black hole formation, and the corresponding scattering cross section

$$b_{max} = 7^{-1/12} \sqrt{\frac{12}{7}} R e^{y_0/R} \left[\frac{R_s}{R e^{y_0/R}} \right]^{1/6} \Rightarrow \sigma_{gauge} = \pi a'^2 \bar{K} \left(\frac{\hat{\alpha}' s}{\alpha'} \right)^{\frac{1}{6}} \quad (2.22)$$

thus $\sigma_{gauge} \sim s^{1/6}$, but it was not clear whether there exists an energy regime corresponding to this behaviour, or whether at E_R the gauge theory goes directly into maximal Froissart behaviour.

3 The “soft Pomeron” in gauge theories; overview of energy regimes

Let us analyze this problem in more detail. We have seen that creating black holes in flat D dimensions generates a gauge theory behaviour $\sigma_{gauge} \sim s^{1/(D-3)}$, specifically $s^{1/2}$ for D=5 and $s^{1/7}$ for D=10. Since it is normal to take D=10, it would be very unlikely to have $s^{1/6}$, corresponding to black hole creation in AdS_5 , as an intermediate behaviour before going over to the $\ln^2 s$ Froissart behaviour. On the other hand, AdS_5 implies D=5, and then we would have $s^{1/2} \rightarrow s^{1/6} \rightarrow \ln^2 s$ which seems OK. But why would then the compact space X_5 be so small as not to be felt at all by the black holes that feel AdS_5 as flat? If however D=10, it seems that the “black holes in AdS_5 ” regime is excluded, so there would be no problem.

But let us think more about what this means. We have seen that most of the gauge theory amplitude (2.2) at large s, fixed t is situated in the IR, near the IR brane, at least if the large \bar{r} behaviour of the wavefunctions is not modified too drastically. But if one can first consider scattering in flat space as a good approximation (as opposed to scattering on the IR brane from the beginning), first as single graviton exchange $\mathcal{A}_{gauge} \sim (\hat{G}s)^2$, then as Regge behaviour $\mathcal{A}_{gauge}^{2 \rightarrow 2} \sim (\hat{\alpha}'s)^{2+\hat{\alpha}'t/2}$, then as black hole creation in flat space, that means the scattering should first feel the curvature of AdS, and just after that to feel only the IR brane.

On the other hand we have seen that black hole creation in just AdS_5 will not do, we get a behaviour $s^{1/7} \rightarrow s^{1/6} \rightarrow \ln^2 s$ that is hard to imagine in the gauge theory. If the compact space X_5 is of size comparable to AdS_5 , when $r \gg R$ the compact space will not be felt, and we get the same inconsistent result.

What about if X_5 has a much larger size? We could say that the wavefunctions $\psi(\bar{r}, \Omega)$ in the IR are such that the average position y_{av} (where most of the AdS scattering takes place in the Polchinski-Strassler formula) is far from the IR brane, and we have a large average size $e^{2y_{av}/R} \bar{R}^2 d\Omega_5^2$ of the compact space. But this will not do, as we can see from (2.21) and (2.22): the effective scale of AdS in the 4d theory is actually $Re^{y_0/R}$, and that is compared to $\bar{R}e^{y_0/R}$, where \bar{R} is the scale of the compact space.

So we need the effective size \bar{R} of the compact space to be much larger than R. That is possible, and would even solve the problem of having most of the scattering in (2.2) away from the IR brane. Indeed, the AdS wavefunctions $\psi(\bar{r})$ are modified at small \bar{r} , but on top of that, AdS space itself will be modified at small \bar{r} , which can be modelled by $\bar{R} = \bar{R}(\bar{r})$. Then as we can see from (2.2), if $\sqrt{g_X} = \bar{R}(\bar{r})^5$ (the volume of the compact space) increases sufficiently with \bar{r} , it will balance the $\bar{r}^{-\beta}$ behaviour of the integral, which drives the bulk of it towards \bar{r}_{min} .

Thus this simple model, that in the IR the effective size \bar{R} of the compact space increases with \bar{r} , can make the average $\langle \bar{r} \rangle$ at which most of the integral is situated to be $\gg \bar{r}_{min}$,

and correspondingly also the effective average size $\langle \bar{R} \rangle$ of the compact space to be $\gg R$. Does this fix our gauge theory contradiction in $\sigma_{tot}(s) : s^{1/7} \rightarrow (s^{1/6?}) \rightarrow \ln^2 s$?

We have analyzed this question in detail in the Appendix. In the case that $\langle \bar{R} \rangle \gg R$, we have an intermediate regime where the compact space can be considered as approximately flat. We have solved the Poisson equation to get the A-S shockwave in $AdS_{d+1} \times X_{\bar{d}}$ in that case, (A.5). At large 4d distances r and $y = y_0$ (y_0 = interaction point), it behaves like

$$\Phi = \frac{K_1 R_s R^n}{r^n} \sim \frac{1}{r^{2(d-1)+\bar{d}}} = \frac{1}{r^{11}} \quad (3.1)$$

Specifically, for $d=4$, $\bar{d} = 5$, we have

$$\Phi(r \gg R, y = 0) = \frac{945}{8} \frac{R_s}{(2\pi)^2} \frac{R^{11}}{r^{11}} \quad (3.2)$$

One then scatters two of these shockwaves and calculates the trapped surface formed in the collision, using the formalism in [23, 7]. One finds a trapped surface that satisfies, at nonzero impact parameter b ,

$$\left(\frac{3\Phi|_{y=0}}{2R}\right)^2 \left(1 - \frac{b^2}{2r^2}\right) = 1 \quad (3.3)$$

and a trapped surface that is there in the absence of the highly curved AdS, and is smaller. One takes the larger trapped surface as describing best the horizon of the black hole formed in the collision, and gets

$$b_{max} = \sqrt{\frac{2n}{n+1}} (n+1)^{-1/n} R \left(\frac{3K_1 R_s}{2R}\right)^{\frac{1}{n}}; \quad R_s = G_4 \sqrt{s} \quad (3.4)$$

thus a cross section $\sigma = \pi b_{max}^2$ and a QCD cross-section which contains a model dependent multiplicative constant, and converts gravity quantities to gauge quantities ($G_4 \rightarrow \hat{G}_4$, $R \rightarrow \Lambda_{QCD}$).

$$\sigma_{gauge} = \bar{K} \pi b_{max}^2 (\hat{\alpha}' s / \alpha')^{1/n} \sim s^{1/n} \quad (3.5)$$

with $n=11$ for $d = 4$, $\bar{d} = 5$.

We see therefore that now we have indeed solved the gauge theory contradiction for the $\sigma_{gauge}(s)$ flattening behaviour. Now we have $s^{1/7} \rightarrow s^{1/11} \rightarrow \ln^2 s$, that is consistent flattening.

So let us review the energy regimes of gauge theories. The gauge theory energy scales are, in increasing order. First, the AdS scale ($1/R$), corresponding to $\hat{E}_{AdS} = \Lambda_{QCD}$. Then, the string scale, corresponding to $\hat{E}_S = \hat{\alpha}'^{-1/2} = \Lambda_{QCD} (g_{YM}^2 N)^{1/4}$. After that, one reaches the Planck scale, corresponding to $\hat{M}_P = N^{1/4} \Lambda_{QCD}$, followed by the correspondence principle scale, at which the string description is replaced by black hole description, corresponding to $\hat{E}_c = \Lambda_{QCD} N^2 / (g_{YM}^2 N)^{7/4}$. Finally, one reaches the scale at which the black hole horizon size equals the AdS size, corresponding to $\hat{E}_R = N^2 \Lambda_{QCD}$.

If the gravity dual would be intrinsically 5d (such that the compact space is always much smaller), one would have two further scales. The scale at which $G_4 s \sim 1$ corresponds in

gauge theory to $\hat{M}_{P,4} = N^{3/8} \Lambda_{QCD}$, and the scale at which $R_s = G_4 \sqrt{s}$ is of the order of R_{AdS} , corresponding in gauge theory to $\hat{E}'_R = N^{3/4} \Lambda_{QCD}$, which is thus \hat{E}_R for pure AdS_5 gravity dual. As we said, the possibility that the dual is always 5d (and the compact space always very small) seems hard to imagine, but is a self-consistent one, so we mentioned it anyway.

Pictorially, one has the energy regimes 0, I, II, III, IV, V:

$$0 \rightarrow |\Lambda_{QCD} \quad I \rightarrow |\hat{E}_s \quad II \rightarrow |\hat{M}_P \quad III \rightarrow (\hat{M}_{P,4}; \hat{E}_c; \hat{E}'_R) \rightarrow |\hat{E}_R \quad IV \rightarrow ? \quad V? \quad (3.6)$$

and we put a question mark because there could be one more scale involved.

Let us explain what happens in each regime. In the regime I, above Λ_{QCD} and before \hat{E}_s , but close to it, in the gravity dual we have single graviton exchange, described by 't Hooft scattering: one particle creates a shockwave, the other moves on a null geodesic. Actually, for this behaviour to be isolated from other behaviours, we need \hat{M}_P close to \hat{E}_s , since as 't Hooft showed, we need actually to have energies close to the Planck scale, not the string scale. We also need to be far away from Λ_{QCD} ($1/R$ in gravity), so that we don't feel the glueball masses (don't feel the AdS curvature in gravity). This is a stringent constraint on the gauge theory, but it could be satisfied in principle. In gauge theory this would also correspond to exchange of a single universal colourless "graviton", which would be a nonrenormalized version of the "Pomeron", with intercept $\alpha(0) = 2$ (graviton).

Further in energy, in regime II, we have Regge behaviour for the string amplitude, and correspondingly Regge behaviour for the gauge theory

$$\mathcal{A}_{gauge}^{2 \rightarrow 2} \sim (\hat{\alpha}' s)^{2 + \hat{\alpha}' t/2} \quad (3.7)$$

Now we have Regge trajectories $\alpha(t) = 2 + \hat{\alpha}' t/2$ replacing the "graviton" exchange.

In regime III, above the Planck scale, we will start producing black holes, corresponding in the gauge theory to nonlinear solitons of the glueball effective field (via the Heisenberg description, extended in this paper to non-saturated behaviour). We will not treat the case of the pure 5d gravity dual. Then, if the dual is 10d, we get $\sigma_{gauge}(s) \sim s^{1/(D_{tot}-3)} = s^{1/7}$, from the decay of the glueball effective field soliton=black hole.

The black holes being created we have argued that happen on the average at a $\langle \bar{r} \rangle \gg \bar{r}_{min}$ for consistency, and thus one doesn't feel the IR brane yet. One first reaches AdS size, when $E = \hat{E}_R$, entering regime IV, and then in $AdS_{d+1} \times X_{\bar{d}}$ one gets

$$\sigma_{gauge} \sim s^{\frac{1}{n}} = s^{\frac{1}{2(d-1)+d}} = s^{\frac{1}{11}} \quad (3.8)$$

As the black holes continue to grow, they will eventually reach the IR brane and grow as large as to be effectively on the IR brane. That should happen at an unknown scale \hat{E}_F , depending on M_1 (the mass of the lightest glueball, and the mass of the lightest KK graviton in the gravity dual), which would signal the onset of the maximal Froissart behaviour in an energy regime V. This scale would depend on the details of the gravity dual.

Up to now we have analyzed the case that the lightest excitation is a glueball. But if the lightest excitation is an almost Goldstone boson like the pion of QCD, the maximal Froissart behaviour will be in terms of the pion field. The simple order-of-magnitude argument of [14]

showed that if the pion is the radion in the RS model, a similar picture emerges, with the IR brane bending under the mass \sqrt{s} . It is not clear how to translate this into a picture involving collision of waves, but it is clear that somehow the bending will become larger first, engulfing the black hole in it. Therefore one will have another unknown scale \hat{E}'_F which will be $\hat{E}_F(M_1 \rightarrow M_L)$ (replace the first glueball mass by the pion mass, or KK graviton mass by the radion mass in the gravity dual), and $\hat{E}'_F < \hat{E}_F$. At that scale, one will have maximal Froissart behaviour in terms of $M_L = m_\pi$.

4 Real QCD and experiments

So what should happen in real QCD? First, one needs an argument that all that we said still applies in QCD. In [23], string corrections to the black hole production via A-S scattering were computed in flat $d=4$, and they were used in [7, 8]. One scattered string-corrected A-S shockwaves and analyzed their effect on the black hole production. The string corrected shockwaves, due to Amati and Klimcik [24] are obtained as follows. One matches the 't Hooft scattering of a superstring in an arbitrary shockwave profile Φ , given by $S = e^{ip^+\Phi}$, with a resummed eikonal superstring calculation [25], $S = e^{i\delta} = \sum_h (i\delta)^h/h! \sim \sum_h (g_s)^h (a_{tree})^h/h!$ (h = loop number). By equating the two results, one gets

$$\Phi(y) = -q^v \int_0^\pi \frac{4}{s} : a_{tree}(s, y - X^d(\sigma_d, 0)) : \frac{d\sigma_d}{\pi} \quad (4.1)$$

where $b \equiv x^u - x^d$ is the impact parameter, that becomes the variable y . This procedure contains both α' and g_s corrections. The g_s corrections come from the eikonal resummation, obtained by gluing tree amplitudes into “ladder diagrams”. The α' corrections come from the fact that Φ at large y is Aichelburg-Sexl + α' corrections (from a_{tree}). When scattering two string-corrected AK shockwaves, one obtains a maximum impact parameter [23]

$$B_{max} = \frac{R_s}{\sqrt{s}} (1 + e^{-\frac{R_s^2}{8\alpha' \log(\alpha' s)}}) \quad (4.2)$$

when the exponent is large in absolute value, and the uncorrected term is the A-S result. The condition for the exponent to be large, when one dimensionally reduces to 4d, gives

$$G_4 s \frac{G_4/\alpha'}{\log(\alpha' s)} > 1 \quad (4.3)$$

or $\sqrt{s} > E_0$, and replacing the formulas that we have in our case gives a QCD energy scale

$$E_0 \sim \frac{\hat{M}_P^2}{\hat{E}_s} \sim \Lambda_{QCD} N^{1/4} g_{YM}^{-1/4} \quad (4.4)$$

As the scattering was not done in our case, we can't trust E_0 , but in any case we see that it is most likely smaller than \hat{E}_R , the scale of the onset of the “soft Pomeron” behaviour $\sigma_{QCD} \sim s^{1/n}$. Why are the calculated string corrections exponentially small? We have no

good physical argument for it, other than the argument 't Hooft gives for the predominance of 't Hooft scattering over any massive interaction corrections: massive interactions are finite range, and they get infinitely time delayed due to the divergence of Φ at small r . But this argument, extended to the black hole production case, is not completely similar, since for instance it was also found in [23] that when the exponent in (4.2) is small in absolute value, we get very large (positive, i.e. increasing B_{max}) corrections.

In the gauge theory, string α' and g_s corrections translate into $1/N$ and $1/(g_{YM}^2 N)$ corrections, thus if string corrections are small, we can apply our calculations in real QCD ($N = N_c = 3$ and $g_{YM} \sim 1$) as well.

So we know that string corrections to the scattering will be insignificant at $s \rightarrow \infty$, and most likely also above \hat{E}_R . But there will be of course corrections to the gravity dual itself. However, as we have not used any details of the gravity dual other than the scales, we know that at most, we can have renormalization of the energy scales, e.g. $R^{-1} \leftrightarrow \Lambda_{QCD}$, $M_P \leftrightarrow \hat{M}_P = N^{1/4} \Lambda_{QCD}$ and $E_R \leftrightarrow \hat{E}_R = N_c^2 \Lambda_{QCD}$.

In QCD, the relevant energy scales are as follows. We have the pion mass $m_\pi \simeq 140 MeV$ that will appear in the Froissart bound (1.1). The mass of the lightest glueball is not known, as it was not discovered yet. On the lattice, one finds $M_1 \sim 1.6 GeV$ [26], and experimental candidates range from $0.6 GeV$ to $1.7 GeV$. Overall, $\Lambda_{QCD} \equiv M_1 \sim 1 GeV$, thus $\hat{E}_R = N_c^2 \Lambda_{QCD} \sim 10 GeV$. We cannot be exact here, as factors of two, as well as the renormalization of energy scales could modify this result. Then also $\hat{M}_P = N_c^{1/4} \Lambda_{QCD} \simeq \hat{E}_s = (g_{YM}^2 N)^{1/4} \Lambda_{QCD} \simeq \Lambda_{QCD}$.

So what do we expect to happen from the analysis in the previous section? The regimes I and II are practically nonexistent. However, Regge behaviour will be present in the elastic ($2 \rightarrow 2$) part of the amplitude, even in the region III, and is indeed observed.

In region III, that is from about $\hat{M}_P \sim 1 - 2 GeV$ to about $\hat{E}_R \sim 10 GeV$, we would expect to create black holes in flat space in the gravity dual, with $\sigma_{QCD,tot} \sim s^{1/7} \simeq s^{0.143}$, but it is not clear that the energy regime is large enough. Also, as we have seen, in this region we could still maybe have large string corrections to the scattering, so our analysis is not guaranteed here.

In region IV, above $\hat{E}_R \sim 10 GeV$, we expect to go over to the “soft Pomeron” behaviour, where we create black holes in $AdS_{d+1} \times X_{\bar{d}}$, and get $\sigma_{QCD,tot} \sim s^{1/n} = s^{1/[2(d-1)+\bar{d}]} = s^{1/11}$.

Since in QCD $m_\pi \simeq 140 MeV < M_1 \sim 1 GeV$, the maximal Froissart behaviour will be in terms of m_π , dual to something like the radion mass M_L . It should onset at an unknown energy scale $\hat{E}'_F(m_\pi)$, depending on the details of the gravity dual.

On the experimental side, as we said in the introduction, in [2] (cited in [3]), one found the “soft Pomeron” behaviour (1.2), where $\sigma_{tot} \sim s^{0.093(2)}$, more precisely $\sigma_{tot} \sim s^\epsilon$, with $\epsilon = 0.0933 \pm 0.0024$, and this behaviour fits well all data above $\sqrt{s}_{min} = 9 GeV$, with a $\chi^2/d.o.f. = 1$ (if one tries to extend this behaviour to lower energies, χ^2 increases). Thus we have remarkable agreement with our predicted soft Pomeron behaviour!

Strangely though, later it was found [4] (cited in [5]) that a statistically better fit (with $\chi^2/d.o.f. = 0.971$) is given by a maximal Froissart behaviour, plus a reaction dependent constant term (1.3), in which case the fit can be extended down to $\sqrt{s}_{min} = 5 GeV$.

In light of our analysis, we can give two possible explanations. One is that the later fit

is a coincidence, which would be supported by the fact that one extends the fit down in energies, not up, which seems counterintuitive. Also, the reaction dependent constant term is maybe less motivated and indicative of a coincidence due to having many parameters in the fit (note though that both the “soft Pomeron” and the later fit have the same number of parameters). Finally, the coefficient B of $\ln^2(s/s_0)$ is only about $\sim 0.31mb \ll \pi/m_\pi^2 = 60mb$, with constant term $Z_{AB} \sim 18-65mb$, compared to $X_{AB} \sim 10-35mb$ for the soft Pomeron fit. Even though B should be $< \pi/m_\pi^2$, Heisenberg’s model gave saturation also for the coefficient, so we would expect B at least to be close to $60mb$, while one finds that instead Z_{AB} reaches up to $65mb > 60mb$. On the other hand, for the soft Pomeron one has $X_{AB} \sim 10-35mb$, so maybe only when $X_{AB}(s/s_0)^\epsilon$ reaches $\sim 60mb$ we will turn over to the Heisenberg (maximal Froissart) behaviour.

Another possibility is that in our analysis, due to the fact that $M_L = m_\pi$ is so small compared to Λ_{QCD} , we have \hat{E}'_F (the onset of maximal Froissart behaviour in m_π) is lower or of the order of \hat{E}_R , even though as we explained, $\hat{E}_F > \hat{E}_R$ (the maximal Froissart behaviour in M_1 should onset after the $s^{1/11}$ behaviour. So one creates black holes in almost flat space, but then the bending of the IR brane catches up before the black holes can feel the curvature of AdS. Thus it could be that both the “soft Pomeron” behaviour $s^{1/11}$ and the maximal Froissart behaviour in m_π coexist, and so the real behaviour would be

$$\begin{aligned}\sigma_{AB} - Y_{1AB}(s_1/s)^{\eta_1} + Y_{2AB}(s_1/s)^{\eta_2} &= X_{AB}(s/s_0)^\epsilon + B \log^2(s/s_0) \\ \sigma_{\bar{A}B} - Y_{1AB}(s_1/s)^{\eta_1} - Y_{2AB}(s_1/s)^{\eta_2} &= X_{AB}(s/s_0)^\epsilon + B \log^2(s/s_0)\end{aligned}\quad (4.5)$$

It would be hard to imagine though that one will then still get the same remarkable agreement with $\epsilon = 0.0933 \pm 0.0024$ onsetting at $9GeV$.

In either case, we would suggest that there is need for a further experimental work, as most of the data is situated around 10 GeV, and further up around 1 TeV, and not much in between.

If indeed one observes the soft Pomeron behaviour, either by itself as in (1.2) or together with the maximal Froissart behaviour as in (4.5), we have an experimental test of string theory, literally counting the extra dimensions, since as we have seen one has $\sigma \sim s^{1/n}$ and n comes from the behaviour of the Laplacean on $AdS_{d+1} \times X_{\bar{d}}$ as $1/r^n = 1/r^{2(d-1)+\bar{d}}$.

Of course it would be nice to find the real gravity dual to QCD, so that one can compute precisely the scales Λ_{QCD} , \hat{M}_P , \hat{E}_R , \hat{E}_F , \hat{E}'_F in particular, and distinguish between the various possibilities. It would also be nice to have a scattering description for the pion Heisenberg (maximal Froissart) behaviour, dual to brane bending. Short of that, we have presented a consistent analysis, and shown that the soft Pomeron behaviour can count the extra dimensions of string theory, and it corresponds in QCD to creation of an effective field theory “soliton”-like structure that then decays, mapped to black hole production in the gravity dual.

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Appendix A. Solution for Φ and trapped surfaces.

In this Appendix we solve for the A-S solution in the $AdS_5 \times X_5$ background (with X_5 very large, almost flat), and calculate the trapped surface that forms in the collision of two such shockwaves.

The equation we need to solve is the Poisson equation in the background,

$$\Delta\Phi = -16\pi G_{d+\bar{d}+1} p \delta^{d-2}(x^i) \delta(y-y_0) \delta^{\bar{d}}(z_i) \quad (\text{A.1})$$

where Δ is the laplacean for massless particles propagating in the $AdS_{d+1} \times X_{\bar{d}}$ background, i.e.

$$\Delta = \nabla_x^2 + e^{-2y/R} (\partial_y^2 - \frac{d}{R} \partial_y) + e^{-2y/R} \partial_{z^i}^2 \quad (\text{A.2})$$

Here x^i are d-2 flat transverse coordinates on the brane, y is the AdS radial coordinate and z^i are \bar{d} (almost) flat coordinates on $X_{\bar{d}}$.

A Fourier transform

$$\phi(q, \bar{q}, y) = \int d^{d-2} x e^{-iq \cdot x} \int d^{\bar{d}} z e^{-i\bar{q} \cdot z} \Phi(x, y, z) \quad (\text{A.3})$$

turns the equation into (outside $y = y_0 = 0$. We choose $y_0 = 0$ for simplicity, as it can be simply reintroduced by rescaling)

$$\phi''(q, \bar{q}, y) - \frac{d}{R} \phi'(q, \bar{q}, y) - (q^2 e^{2y/R} + \bar{q}^2) \phi(q, \bar{q}, y) = 0 \quad (\text{A.4})$$

We see that for $y/R \ll 1$, the equation is the same as the equation for AdS_{d+1} found in [23], in variables $Q^2 = q^2 + \bar{q}^2$. Therefore the solution for $y/R \ll 1$ is

$$\begin{aligned} \Phi(r, y) &= \frac{4G_{d+\bar{d}+1} p e^{\frac{dy}{2R}}}{2\pi} \int \frac{d^{d-2} q}{(2\pi)^{d-2}} e^{i\vec{q} \cdot \vec{x}} \int \frac{d^{\bar{d}} \bar{q}}{(2\pi)^{\bar{d}}} e^{i\bar{q} \cdot z} K_{d/2}(e^{y/R} RQ) I_{d/2}(RQ) \\ &= \frac{4G_{D+1} p e^{\frac{dy}{2R}}}{(2\pi)^{D+1}} \frac{1}{r^{\frac{D-4}{2}}} \int_0^\infty dQ Q^{\frac{D-2}{2}} J_{\frac{D-4}{2}}(Qr) K_{d/2}(e^{y/R} RQ) I_{d/2}(RQ) \\ &= \frac{\bar{C} e^{\frac{dy}{2R}}}{r^{D-2}} \int_0^\infty dw w^{\frac{D-2}{2}} J_{\frac{D-4}{2}}(w) K_{d/2}(e^{y/R} \frac{Rw}{r}) I_{d/2}(\frac{Rw}{r}) \end{aligned} \quad (\text{A.5})$$

where $r^2 = \vec{x}^2 + \vec{z}^2$, $D = d + \bar{d}$ and $\bar{C} = 8G_{D+1} l p / (2\pi)^{(D-4)/2}$, and the solution is valid for $y > 0$. If $y < 0$, we should exchange $K_{d/2}$ and $I_{d/2}$. We are interested in the behaviour of Φ at large r/R , thus for small argument of $K_{d/2}$ and $I_{d/2}$.

When we expand $K_{d/2}(aw) I_{d/2}(bw)$ at small w we obtain mostly w^{2n} terms, but they will give a zero result after integration, since

$$\int_0^\infty dw w^{D/2-1+2n} J_{D/2-2}(w) = 0 \quad (\text{A.6})$$

The first nonzero result comes when we get a $w^n \log(w)$ term. We can check that the first such term in the expansion of $K_{d/2}(aw)I_{d/2}(bw)$ is $\sim f a^{d/2} b^{d/2} w^d \log(w)$, (f=numerical constant) and this gives a nonzero integral.

Thus the behaviour of Φ is

$$\Phi(r \gg R, y = 0) = \frac{K_1 R_s R^{D+d-2}}{r^{D+d-2}} = \frac{const.}{r^{2(d-1)+\bar{d}}} \quad (\text{A.7})$$

where

$$K_1 = \frac{f}{(2\pi)^{(D-4)/2}} \int_0^\infty dw w^{d+\frac{D-2}{2}} J_{\frac{D-4}{2}}(w) \log(w) \quad (\text{A.8})$$

In the following we restrict to the physical case $d = 4, \bar{d} = 5$, but present some formulas at general d, \bar{d} . The calculation mirrors exactly the one in Appendix A of [7].

We have

$$I_2(bx)K_2(ax) = \frac{b^2}{4a^2} + ct.x^2 + ct.x^4 - \frac{1}{64}a^2b^2x^4 \log(x) + o(x^5) \quad (\text{A.9})$$

and using

$$I = \int_0^\infty dw w^{4+7/2} J_{5/2}(w) \log(w) = -3780\sqrt{2\pi} \quad (\text{A.10})$$

we get

$$\Phi(r \gg R, y = 0) \simeq \left(-\frac{I}{64}\right) \frac{\bar{C} R^4}{r^{11}} = \frac{945\sqrt{2\pi}}{16} \frac{\bar{C} R^4}{r^{11}} \quad (\text{A.11})$$

and

$$\Phi(r \gg R, y/R \ll 1) \simeq \frac{\bar{C} e^{2y/R}}{r^7} \int_0^\infty dw w^{7/2} J_{5/2}(w) K_2(e^{y/R} \frac{Rw}{r}) I_2(\frac{Rw}{r}) \quad (\text{A.12})$$

Using $wK'_\nu(w) + \nu K_\nu(w) = -wK_{\nu-1}(w)$, we get

$$\partial_y \Phi = -\frac{\bar{C} e^{2y/R}}{Rr^7} \int_0^\infty dw w^{7/2} J_{5/2}(w) (e^{y/R} \frac{Rw}{r}) K_1(e^{y/R} \frac{Rw}{r}) I_2(\frac{Rw}{r}) \quad (\text{A.13})$$

Then

$$\partial_y \Phi|_{y=0} = -\frac{\bar{C}}{r^8} \int_0^\infty dw w^{9/2} J_{5/2}(w) K_1(\frac{Rw}{r}) I_2(\frac{Rw}{r}) \quad (\text{A.14})$$

and

$$\partial_y^2 \Phi|_{y=0} = \frac{2}{R} \partial_y \Phi|_{y=0} + \frac{\bar{C}}{r^9} \int_0^\infty dw w^{11/2} J_{5/2}(w) K_0(\frac{Rw}{r}) I_2(\frac{Rw}{r}) \quad (\text{A.15})$$

Expanding the Bessel functions at small argument

$$\begin{aligned} K_0(x)I_2(x) &\simeq \left(-\frac{\gamma}{8} + \frac{\log(2)}{8} - \frac{\log(x)}{8}\right)x^2 + o(x^4) \\ K_1(x)I_2(x) &\simeq \frac{x}{8} + \left(-\frac{1}{48} + \frac{\gamma}{16} - \frac{\log(2)}{16} + \frac{\log(x)}{16}\right)x^3 + o(x^5) \end{aligned} \quad (\text{A.16})$$

we get

$$\begin{aligned}\partial_y \Phi|_{y=0} &\simeq \left(-\frac{I}{16}\right) \frac{\bar{C} R^3}{r^{11}} = \frac{4\Phi|_{y=0}}{R} \\ \partial_y^2 \Phi|_{y=0} &\simeq \frac{8}{R^2} \Phi|_{y=0} - \frac{I}{8} \frac{\bar{C} R^2}{r^{11}} = \frac{16\Phi|_{y=0}}{R^2}\end{aligned}\tag{A.17}$$

The trapped surface condition is now

$$(\partial_i \Psi)^2 + e^{-2y/R} (\partial_y \Psi)^2 + e^{-2y/R} (\partial_\mu \Psi)^2 = 4; \quad \Psi = \Phi + \zeta \tag{A.18}$$

where $\mu = 1, 5$ are indices for the z^μ coordinates on X_5 . We work at $z=0$ (fixed position in the extra dimensions), thus $(\partial_\mu \Phi)^2|_{z=0} = 0$.

The above condition is matched against $\Psi = C = \text{const.}$ to find ζ perturbatively, $\zeta = \zeta_0(r) + \zeta_1(r) + \zeta_2(r)y^2/2$, with $\zeta_0 = 0$ for consistency. Then

$$\Psi = f + ay + \frac{y^2}{2}g + \dots \tag{A.19}$$

where

$$\begin{aligned}f &= \Phi|_{y=0} = (-I/64) \frac{\bar{C} R^4}{r^{11}} = \frac{K_1 R_s R^n}{r^n} \\ a &= \partial_y \Phi|_{y=0} + \zeta_1 = \frac{4}{R} f + \zeta_1 \\ g &= \partial_y^2 \Phi|_{y=0} + \frac{d}{R} \zeta_1 = \frac{16f}{R^2} + \frac{4\zeta_1}{R}\end{aligned}\tag{A.20}$$

If a is nonzero, one then has to match

$$\begin{aligned}4 &= f'^2 + a^2 + y(2a'f' - \frac{2a^2}{R} + 2ag) + \dots \\ \text{vs. } C &= f + ay\end{aligned}\tag{A.21}$$

If $\zeta_1 = 0$ one obtains

$$\begin{aligned}1 &= \left(\frac{2f}{R}\right)^2 \left[1 + \frac{6y}{R} + \left(\frac{nR}{4r}\right)^2 + \dots\right] \\ \text{vs. } C^2 &= f^2 \left[1 + \frac{8y}{R}\right]\end{aligned}\tag{A.22}$$

which has no solution, thus one needs to take a nonzero ζ_1 . If one takes ζ_1 and a of the same order, giving $a = -\alpha f/R$, we get

$$\begin{aligned}1 &= \left(\frac{\alpha f}{2R}\right)^2 \left[1 + \frac{6y}{R} + \left(\frac{nR}{\alpha r}\right)^2 + \dots\right] \\ \text{vs. } C^2 &= f^2 \left[1 - \frac{2\alpha y}{R} + \dots\right]\end{aligned}\tag{A.23}$$

which has the solution $\alpha = -3$. Thus we have a trapped surface with

$$a = \frac{3f}{R} \quad (\text{A.24})$$

which gives the condition for the size of the trapped surface

$$\frac{3f}{2R} = 1 \quad (\text{A.25})$$

and at nonzero impact parameter of the two colliding A-S shockwaves we get approximately

$$\left(\frac{3f}{2R}\right)^2 \left(1 - \frac{b^2}{2r^2}\right) = 1 \quad (\text{A.26})$$

With the definitions

$$f = \frac{K_1 R_s R^n}{r^n}; \quad \left(\frac{3K_1 R_s R^n}{2R}\right)^2 = a; \quad r^2 = x \quad (\text{A.27})$$

one has to solve the equation

$$g(x) = x^{n+1} - ax + \frac{ab^2}{2} = 0 \quad (\text{A.28})$$

The maximum b for which there is a solution is found from $g'(x_0) = 0, g(x_0) = 0$, giving

$$b_{max}^2 = \frac{2n}{n+1} \left(\frac{a}{n+1}\right)^{\frac{1}{n}} \Rightarrow b_{max} = \sqrt{\frac{2n}{n+1}} (n+1)^{-1/n} R \left(\frac{3K_1 R_s}{2R}\right)^{\frac{1}{n}} \quad (\text{A.29})$$

As in [7] one finds an extra trapped surface that would be there in the absence of AdS, and that surface is smaller. The same physical argument, that the large warping of AdS is expected to create a larger black hole, applies. Therefore, one will take the above trapped surface solution as describing best the horizon of the formed black hole.

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